LSU Dept. of Physics and Astronomy
Qualifying Exam
E&M Question Bank
(11/2025)

1. An insulated disk of radius "a" and infinitesimal thickness carries a positive definite surface charge density "σ".

   a) Calculate the electrostatic potential at the center of the disk.

   b) Calculate the electrostatic potential at an arbitrary point on the axis of symmetry.

   c) Calculate the electrostatic potential at the edge of the disk.

   d) Using the results derived above, deduce the direction of the electric field in the plane of the disk.

2. Consider a non-relativistic particle of mass "m" and charge "e" that is acted on by an external force, "$\vec{F}$". The instantaneous power radiated by the charge is given by the Larmor formula

$$P(t) = \frac{2}{3}\frac{e^2}{c^3}|\vec{a}|^2 \quad .$$

   a) Write down the relativistic generalization of the Larmor Formula.

   b) Express your answer to part a) in terms of the external force $\vec{F}$, assuming that the charged particle is being accelerated in a linear accelerator.

   c) In typical linear accelerators the energy gains are approximately $10 MeV$ per meter. Demonstrate that for relativistic particles the radiation losses are negligible in comparison with the energy gains.

3. Consider a simple model for a dielectric in which the atomic electrons are bound to a fixed site by means of a harmonic restoring force $-\omega_o^2\vec{x}$. Suppose that in addition the electrons are acted upon by a damping force $-\gamma_0\dot{\vec{x}}$. This simple model leads immediately to the following expression for the permittivity $\varepsilon(\omega)$:

$$\varepsilon(\omega) = 1 + \frac{4\pi Ne^2}{m}\frac{1}{\omega_o^2 - \omega^2 - i\omega\gamma_o}$$

where N is the number of electrons per unit volume.

If we consider the limit in which $\omega_o \to 0$, then the atomic electrons are no longer bound to a fixed site, and the simple model reduces to a model for conductivity. Use the expression shown above to derive the conductivity $\sigma(\omega)$ in the limit that $\omega_o \to 0$.

4. A magnetic dipole of moment "$\vec{M}$" is located a distance "d" from the surface of a semi-infinite slab of magnetic material with permeability "μ." The magnetic dipole "$\vec{M}$" is perpendicular to the surface and points toward the surface. Calculate the force exerted on the dipole "$\vec{M}$."

5. Using the method of images, consider the problem of a point charge q <u>inside</u> a hollow, grounded, conducting sphere of inner radius a. Find the potential inside the sphere.

6. Consider the following relativistic transformations.

a) Suppose that the electric field $\vec{E}$, has <u>Cartesian</u> components $E_x,\ E_y,\ E_z$ in an inertial frame K and that the magnetic field is zero. Write down an expression showing how their corresponding values $E'_x,\ E'_y,\ E'_z$ may be calculated in any other inertial frame K'.

b) Consider the special case of an inertial frame K' that is moving in the positive x direction with a velocity v, as viewed from an inertial frame K. The 4-coordinates x^μ in K and $(x')^\nu$ in K' (where, as usual, $\mu,\nu = 0,1,2,3$) are related by a Lorentz transformation matrix. Write down this matrix.

c) For the specific transformation discussed in (b), calculate $E'_x,\ E'_y,\ E'_z$.

7. The plates of a semi-infinite capacitor are maintained at a constant potential difference V by means of a battery. The plates are a distance d apart. A charge q of mass m is released from rest at the surface of one plate and moves toward the other plate under the influence of the electric field. Ignoring complicating effects, such as gravity and image changes,

a) Derive an expression for the power radiated by the charge during its motion.

b) Calculate the total energy radiated by the charge during its motion and compare with the change in kinetic energy.

c) Identify any assumptions that underlie your calculation.

NOTE: The results in (a) and (b) should be expressed in terms of q, m, V, d and c (velocity of light).

8. Consider a general current distribution $\vec{J}(\vec{x}\,')$ localized in a small region of space.

a) Define the associated magnetic moment $\vec{m}$.

b) Write down a general result for the vector potential $\vec{A}(\vec{x})$ in terms of $\vec{J}(\vec{x}\,')$ (1 point) and use it to write down (proof not required) an expression for $\vec{A}(\vec{x})$ in terms of $\vec{m}$ in the case where $|\vec{x}| >> |\vec{x}\,'|$.

c) Assuming that a magnetostatic field is due entirely to a localized distribution of permanent magnetization, show that

$$\int \vec{B}\cdot\vec{H}\ d\vec{x} = 0,$$

where the integral is taken over all space.

9. A plane polarized electromagnetic wave of frequency ω in free space is incident normally on the flat surface of a nonpermeable medium of conductivity σ and dielectric constant ε.

a) Given that the ratio of the amplitudes of the reflected and incident waves is

$$\frac{E_O''}{E_O} = \frac{1-n}{n+1} \quad ,$$

where n is the refractive index, and that we are dealing with a good conductor, derive the Hagen-Rubens relation for the reflectivity.

b) Assuming that the conductivity of sodium is $2.1 \times 10^{1/}\ s^{-1}$, calculate its reflectivity at a wavelength of 10 microns.

10. One of the two cases below is an unphysical electrostatic field. (Do *not* assume that electric charge density is zero.) :

case 1) $\vec{E} = k\left[xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}\right]$

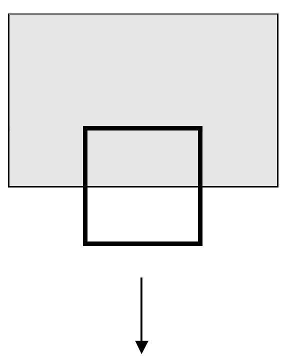
case 2) $\vec{E} = k\left[y^2\hat{\mathbf{x}} + \left(2xy + z^2\right)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}\right]$,

where k is an arbitrary constant with the appropriate units.

a) Show explicitly why each case is a physical or unphysical electrostatic field.

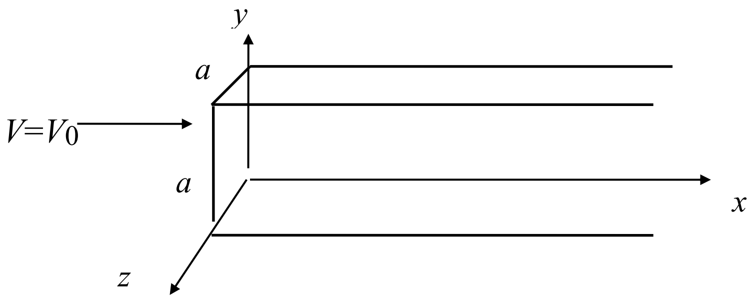
b) For the physical case, find the associated electrostatic potential V, using the origin as the reference point. Check your answer by computing $\vec{\nabla} V$.

11. A square "loop" has been cut out of a sheet of aluminum. It is then placed so that the top portion is in a uniform magnetic field **B** (shaded in the diagram below), and is allowed to fall under gravity.



Calculate the terminal velocity of this loop, in meters/second, given B = 1 Tesla, the resistivity of Aluminum = $2.65 \times 10^{-8}\,\Omega \cdot m$, and $g = 9.8\,m/s^2$.

12. The sides of a half-infinite square metal pipe (sides a) are grounded ($V = 0$). The end, at $x = 0$, is maintained at a specified constant electric potential V_0, as shown in the figure below.



a) Find the potential inside the pipe, for $x \geq 0$. (Hint: the standard solution is an infinite series.)

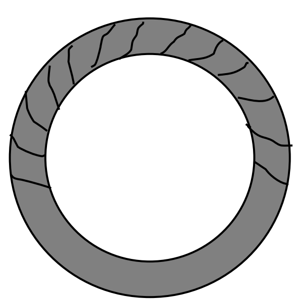
b) Use the result above to find the actual value of the electric field in the middle of the pipe, $y = z = 0.5a$, at a distance $x = 4a$ from the end, where $a = 10^{-2}$ meter and $V_0 = 1$ Volt. (**Hint**: the first two or three terms in the series should provide sufficient accuracy.)

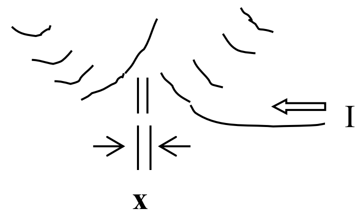
13. Calculate the capacitance C of a spherical capacitor of inner radius R_1 and outer radius R_2 filled with a dielectric given by

$$\varepsilon = \varepsilon_0 + \varepsilon_1 \cos^2 \theta$$

where θ is the polar angle.

14. A toroidal winding of N turns encloses a doughnut-shaped ferromagnetic ring with a small gap of width **x** as shown in the figure. Consider the gap to be sufficiently small so that fringing fields can be neglected. The mean radius of the toroid is **r** and a steady current of **I** flows in the winding. The relative permeability of the ferromagnetic material is $\mathbf{k_m}$ (Note: the permeability is in general a function of the applied field).





a) Determine the ratio of the magnetic field in the gap, $\mathbf{H_g}$, to the magnetic field inside the ferromagnetic material, $\mathbf{H_i}$.

b) Both the field in the gap and the current in the winding can be measured. Develop an expression for the permeability of the material in terms of these measured quantities and the geometry of the toroid.

15. Starting from the Lagrangian for a charged particle in an electromagnetic field

$$\mathbf{L = (1/2)mv^2 - q(V - v \cdot A)}$$

derive the equation of motion for the particle in ordinary Newtonian form.

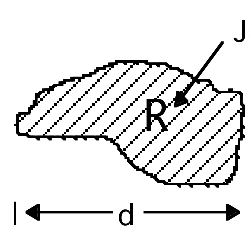
16. A charge Q is situated at (2a, 0, 0) and a charge $2Q$ is situated at (-a, 0, 0).

a) Identify every point in space where the electric field E is zero.

b) Consider two concentric metal (conducting) spheres, of radii $\mathbf{R_1}$ and $\mathbf{R_2}$ carrying total charges $\mathbf{Q_1}$ and $\mathbf{Q_2}$ respectively. What is the potential of the outer sphere, the inner sphere and inside inner sphere .

17. The vector potential for a given current distribution with a sinusoidal time dependence $\boldsymbol{J}(\boldsymbol{x},t) = \boldsymbol{J}_\omega(\boldsymbol{x})e^{-\omega t}$ is given by:

$$\boldsymbol{A}_\omega(\boldsymbol{x}) = \frac{1}{c}\int_R \boldsymbol{J}_\omega\ (\boldsymbol{x}')\frac{e^{ik|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|}d^3x' \quad ,$$



where $k = \omega/c$ $\boldsymbol{B}_\omega = \nabla \times \boldsymbol{A}_\omega$, and $\boldsymbol{E}_\omega = \frac{i}{k}\nabla \times \boldsymbol{B}_\omega$.

There are three *length scales* to consider:

1. d: size of the region where $\boldsymbol{J}_\omega(\boldsymbol{x}') \neq 0$.

2. λ: $= 2\pi/k = 2\pi c/\omega$.

3. r: distance of the observation point from the current.

Derive the following electric dipole radiation field expressions *justifying each approximation with relevant conditions e.g. (length scale)₁ << (length scale)₂ etc.*

$$\begin{aligned} \boldsymbol{A} &= -ik\boldsymbol{p}\frac{e^{ikr}}{r}, \\ \boldsymbol{B} &= k^2(\boldsymbol{n}\times\boldsymbol{p})\frac{e^{ikr}}{r}, \\ \boldsymbol{E} &= \boldsymbol{B}\times\boldsymbol{n}, \end{aligned}$$

where $\boldsymbol{n} = \boldsymbol{x}/r$, $\boldsymbol{p} = \frac{i}{\omega}\int \boldsymbol{J}d^3x'$.

18. A solenoid contains "N" turns per unit length of perfectly conducting (R=0) wire. The cross sectional area of each loop in the solenoid is "A," and the length of the solenoid is "ℓ." The core of the solenoid is filled with a linear, isotropic, homogeneous material of magnetic susceptibility μ. The ends of the solenoid are connected to a source of variable EMF, V(t), that is adjusted to produce the current specified below:

$$i(t) = \begin{cases} 0 & \text{if } t \le 0; \\ \mathrm{I(t)} & \text{if } 0 \le t \le \tau \text{ with } I(\tau) = I_0; \\ I_0 & \text{if } \tau \le t; \end{cases}$$

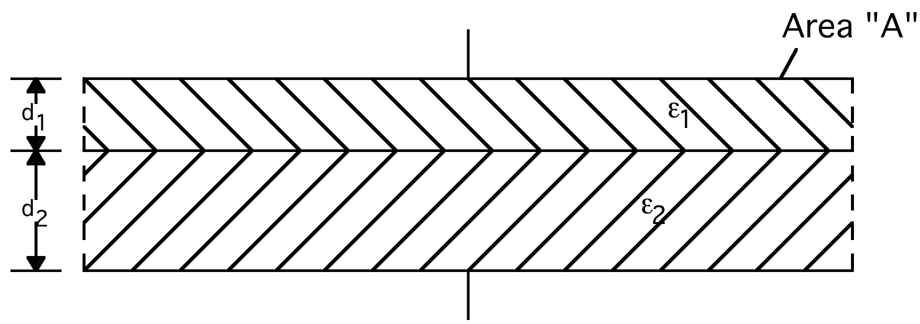
Ignoring all end effects,

a) Explain why the source of EMF must do work despite the fact that the wire is perfectly conducting, meaning that there is no loss of energy due to Joule heating.

b) Calculate V(t) explicitly.

c) Specifically using your answer to part (b), calculate the total work done by the battery and identify that part of the work that is stored in the magnetization of the core.

19. A parallel plate capacitor has a composite dielectric filling as shown in the figure.



a) Derive an expression for its capacitance.

b) A parallel plate capacitor is formed with two metal plates 10 cm by 10 cm separated by an air space 4 mm wide. The capacitor is charged from a 400 V supply. Then, the supply is disconnected and immediately a sheet of glass 10 cm by 10 cm by 2 mm is placed midway between the plates.

i) Calculate the capacitance of the capacitor with and without the glass, for which $\varepsilon_2 = 6\varepsilon_0$.

ii) Calculate the potential difference across the capacitor after inserting the glass.

iii) Calculate the stored energy, before and after inserting the glass plate and explain any difference.

20. The electric potential just outside of the surface of a dielectric sphere of radius a, dielectric constant ε, and fixed surface charge density σ is

$$\phi = A \; Y_{2,0}(\theta,\phi) = A \; \sqrt{\tfrac{5}{16\pi}}\,(3\cos^2\theta - 1)$$

a) Find the electric potential everywhere outside the sphere.

b) Find the electric potential everywhere inside the sphere.

c) Determine the surface charge density σ.

21. Answer briefly and to the point.

a) What is Cherenkov radiation and what condition is necessary to obtain it?

b) What is Thomson scattering? How does it differ from Compton scattering?
c) What is the difference, if any, between cyclotron radiation and synchrotron radiation?
d) There have been 3 different frequencies of choice used for transmission of light by optical fibers. What are these frequencies and why were they chosen and in some cases, later fell out of favor?
e) Describe briefly in words the 3 sources of intrinsic losses in optical fibers and the wavelengths region where they predominate.

22. A sphere of radius "R" is constructed of an imperfect conductor; that is, the conductivity σ is a non-zero, non-infinite constant. Initially the sphere is uncharged, but at time t = 0 a charge "Q" is deposited at the center of the sphere.

a) Calculate the rate at which the charge at the center of the sphere decays. Identify the decay constant.

b) Prove that the charge density $\rho(\vec{r},t)=0$ for all $\vec{r}$ that satisfy the inequality $0 < |\vec{r}| < R$.

c) Calculate the rate at which charge accumulates on the outer surface of the sphere.

d) Compare your answers to parts (a) and (c) and comment as necessary.

23. Two charges, +q and –q, are attached to the ends of a rigid rod. The rod rotates about the z-axis with an angular frequency ω.

a) Calculate the radiated power if the two charges are regarded as independent and uncorrelated.
b) Calculate the radiated power if the two charges are regarded as a single indivisible physical system.
c) Compare your answers to parts (a) and (b) and comment as necessary.

24. A long coaxial cable carries current I. The current flows down the surface of the inner cylinder (of radius a) and back along the outer cylinder (of radius b).

a) Calculate the self-inductance of the cable.

b) Calculate the power (energy per unit time) transported down the wire.

c) Assume the two conductors are held at potential difference V and carry current I in opposite directions. Show that P = IV.

25. Suppose you had an electric charge q_e and a magnetic monopole q_m. The field of the electric charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_e}{r^2}\hat{r} \ ,$$

and the field of the magnetic monopole is

$$B = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{r} \quad .$$

Find the total angular momentum stored in the fields, if the two charges are separated by a distance d.

26. An infinite straight wire lies along the z-axis. The wire is neutral and carries a current that increases linearly in time:

$$I(t) = \begin{cases} 0 \ for\, t < 0 \\ kt \ for\, t \geq 0 \end{cases} \quad where \ \ k > 0 \ .$$

What are the fields E(t) and B(t) at a distance d from the wire?

Hint : $$\int \frac{dz}{\sqrt{d^2 + z^2}} = \ln\left(z + \sqrt{z^2 + d^2}\right).$$

27. Why is the sky blue? Part of the answer is as follows:

a) Sunlight passing through the atmosphere causes individual atoms to oscillate like dipoles. The oscillating dipoles absorb the incident light and re-emit it. If the amplitude of the dipole displacement from equilibrium is x_o under the influence of light of frequency $\nu = \omega/2\pi$, what is the power of the re-radiated light from a single dipole?

b) If the incident solar radiation had equal intensity in the blue and red regions of the spectrum, what would be the ratio of re-emitted blue light to re-emitted red light?

28. A π meson (m_1c^2 = 140 MeV) collides with a stationary target proton (m_2c^2 = 938 MeV) to create a K meson (m_3c^2 = 494 MeV) and a lambda hyperon (m_4c^2 = 1115 MeV).

a) Find the kinetic energy in MeV of the incident π meson at threshold for production of K mesons.
b) Express the kinetic energy of the π meson in terms of the K meson when the latter is created at 90° in the laboratory.
c) Find the kinetic energy of the K mesons at 90° in the lab when the incident π has a kinetic energy of 1500 MeV.

29. Inertial system S' moves at constant velocity $\mathbf{v} = \beta c(\cos\varphi\ \hat{x} + \sin\varphi\ \hat{y})$ with respect to inertial frame S. Their axes are parallel, and their origins coincide at t = t' = 0. Find the Lorentz transformation matrix from S to S'.

30. The x-y plane forms the boundary between two non-magnetic linear media, medium 1 to the left with index of refraction n_1, and medium 2 to the right with index n_2. A plane wave traveling in the z direction is polarized along the x axis. What is the ratio of reflected intensity to incident intensity, and what is the ratio of transmitted intensity to incident intensity?

31. In the Bohr hydrogen atom, the electron in its ground state travels in a circle of radius 5×10^{-11} m, held in orbit by the Coulomb attraction of the proton. The mass of the electron is 9.11×10^{-31} kg, the electron charge is 1.6 $\times 10^{-19}$ C, and $\varepsilon_o = 8.85 \times 10^{-12}$ C/N m^2.

a) Show that the orbital velocity of the electron is non-relativistic.

b) According to classical electrodynamics, the electron should radiate and hence spiral in to the nucleus. Calculate the lifespan of the Bohr atom.

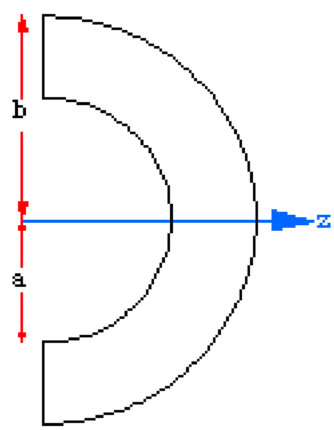
32. a) Sketch lines of $\vec{B}$ and lines of $\vec{H}$ for a magnetized bar and compare with those for a solenoid.

b) Given two iron bars, identical in appearance, one magnetized, the other not, tell how to distinguish them without using external magnetic fields. (You are allowed to measure forces)

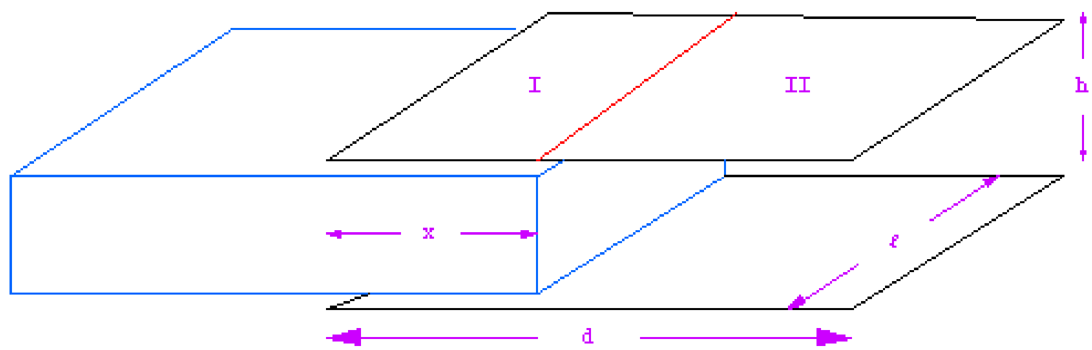
33. a) A sphere of radius a carries a uniform surface charge distribution σ. The sphere is rotated about a diameter with constant angular velocity ω. Find the vector potential and the magnetic – flux density both inside and outside the sphere.

b) A coil is wound on the surface of a sphere such that the field inside the sphere is uniform. What is the winding? (This form of winding is used in the Westing house-Goudsmit mass spectrometer.)

34. Consider the thick hemispherical shell of inner radius a and outer radius b, shown in cross section in the accompanying figure. If the shell is uniformly magnetized along its axis of symmetry (z – axis of the diagram), show that a small compass needle placed at the origin will swing freely.



35. Consider a rectangular piece of dielectric that fills the space between the plates, partially inserted into a plane – parallel condensor. What are the energies of the system as a function of position, and hence the forces drawing the dielectric into the condensor, if the condensor plates are (a) insulated or (b) held at constant potential?



36. A spherical conducting shell occupies the region a < r < b. Find the electric field everywhere, assuming that:

a) A point charge q is in the interior (r < a) of the shell at a distance d < a from the center.

b) A point charge q is outside of the shell at a distance d > b from the center.

c) A point outside a hollow conducting sphere will produce no field in the hollow interior; the metal "shields" the interior. The solution to (a) shows that a point charge inside the hollow interior will produce a field outside; the same metal ceases to act as a "shield". Explain.

37. Two similar charges are placed a distance 2b apart. A grounded conducting sphere is place midway between them. What radius (approximated within 1%) must the conducting sphere have in order to neutralize the mutual repulsion of the two charges?