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Abstract

It is well-known that real wages are procyclical conditional on a monetary policy shock. This paper challenges this conventional view by using a quantitative heterogeneous-agent New Keynesian economy with sticky wages. In the model with benchmark calibration, the wage rate per effective unit of labor decreases following a monetary expansion, while the data-consistent real wages (average hourly earnings) increase. This implies that true real wages may be countercyclical conditional on a monetary policy shock, but the data may predict the wrong direction of real wages due to the inconsistent definition. Therefore, the predictions of New Keynesian models with wage rigidities are consistent with the data.

JEL classification: E52, J31, D31

Keywords: Monetary policy; Real wages; Labor share; Earnings inequality

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1 Introduction

Are real wages procyclical in response to a monetary policy shock? This is a fundamental question in monetary policy analysis since real wages are one of the key drivers of the monetary transmission in New Keynesian models. As is well-known, the labor share plays a vital role in the transmission mechanism of monetary policy in standard New Keynesian economies,[1] and real wages are a crucial component of the labor share.[2] It is, therefore, important that structural models used for monetary policy analysis generate the cyclical behavior of the labor share and its components in ways that are consistent with the data. In this sense, some have criticized the monetary transmission channel in canonical *sticky-price* New Keynesian macroeconomic models, arguing that they fail to match the cyclicality of the labor share. These models predict the procyclical labor share (or equivalently countercyclical markups or profits) conditional on monetary policy shocks,[3] while empirical evidence shows that the labor share decreases following a monetary expansion (e.g., Cantore, Ferroni and Leon-Ledesma (2020) and Nekarda and Ramey (2020)).[4] In response to the perceived mismatch between the data and the predictions of the standard sticky-price model, some recent work, such as Nekarda and Ramey (2020), among others, suggests that New Keynesian models may benefit from refocusing on sticky wages rather than sticky price. Indeed, several recent studies advocate such a shift and find that models with wage rigidities exhibit more plausible responses of output, prices, and profits to monetary policy shocks from both quantitative and qualitative perspectives (e.g., Broer et al. (2019) and Hagedorn et al. (2019)).

One of the critical limitations in the models with a standard sticky wages setting is that this class of models fails to replicate the joint behavior of the labor share and its components conditional on a monetary policy shock (Cantore, Ferroni and Leon-Ledesma, 2020). For example, for the model economy with wage rigidities to generate the countercyclical labor share, real wages should decrease following a monetary expansion. In contrast to the prediction of these models, many papers in empirical literature provide robust evidence that the real wage rate increases following a monetary expansion (Christiano, Eichenbaum and Evans, 2005; Coibion et al., 2017; Cantore, Ferroni and

[1] Note that, in the simplest version of the New-Keynesian model, the labor share is equal to the inverse of the markup of price over marginal cost.

[2] In general, the labor share can be broken down into real wages and labor productivity.

[3] Specifically, in these models, an expansionary monetary policy should increase the labor share since nominal prices cannot fully adjust to increased aggregate demand.

[4] Cantore, Ferroni and Leon-Ledesma (2020) empirically document that a monetary policy tightening greatly induces an increase in the labor share in the U.S., the Euro area, the U.K., Australia, and Canada. Nekarda and Ramey (2020) also present evidence that the price markup, which is based on the inverse of the labor share, increases in response to expansionary monetary policy shocks.

Leon-Ledesma, 2020). This paper revisits the cyclicality of real wages conditional on monetary policy shocks and challenges the conventional view by developing a quantitative heterogeneous-agent New Keynesian economy. The key finding, in short, is that actual real wages may be countercyclical conditional on a monetary policy shock. The disparity between data and theory is not from the model but a different definition of real wages in the data, so there is no puzzling mismatch if the right measure for real wages is used.

The model economy in this paper builds on a heterogeneous-agent New Keynesian model with both nominal price and wage rigidities. The model is formulated as an incomplete market model following Huggett (1993) and Aiyagari (1994): households cannot fully insure against individual productivity shocks. The market incompleteness, together with borrowing constraints, generates substantial heterogeneity across households and leads to individual households' different responses to monetary policy shocks. For the remaining part of the model, standard assumptions in the New Keynesian literature are used. It is assumed that both nominal prices and wages are sticky, and there are monopolistically competitive goods and labor markets. The government plays a role in collecting taxes from households and supplying public bonds to asset markets. Finally, the central bank conducts monetary policy by following a conventional Taylor rule.

In the model economy, the real wage rate, w, is defined as the wage rate per effective labor (N). Effective labor is not observable in practice, so the model-consistent real wage rate is difficult to identify. In other words, one may observe total labor compensation, wN, but cannot identify w and N separately. For this reason, an alternative measure is used in practice: real wages in the data (denoted by ω, hereafter) are generally defined as "average hourly earnings" (i.e. total labor compensation divided by total hours worked):

$$\omega = \frac{wN}{H}, \tag{1}$$

where H is total hours worked. Accordingly, the definition of real wages in the data is inconsistent with that in the model (i.e., $\omega \neq w$). Moreover, due to rich heterogeneity in the economy, dynamics of w and ω may be very different *quantitatively* and/or *qualitatively*.[5] Hence, it is crucial to ask how the data-equivalent real wages in the model respond to a monetary policy shock.

I find that the heterogeneous-agent New Keynesian model with wage rigidities can jointly match

[5] In contrast, in the representative-agent model, where there are infinitely many identical households, the aggregate hours' dynamics will be precisely the same as that of effective labor, so ω always has the same dynamics as w.

the cyclical behavior of the labor share and data-consistent real wages conditional on monetary policy shocks. The perceived disparity between the data and theory arises due to an inconsistent measure for real wages in the data. In the model with benchmark calibration, the real wage rate per effective unit of labor decreases following a monetary expansion while the data-consistent measure for real wages—average hourly earnings—increases. Therefore, I argue that true real wages may be countercyclical conditional on a monetary policy shock. Hence, the predictions of New Keynesian models that incorporate sticky wages are consistent with the data. In this sense, this finding can back up the recent discussions of the importance of wage rigidity in the transmission of monetary policy (Broer et al., 2019; Nekarda and Ramey, 2020).

The question is, then, what explains procyclical real wages in the data? I find that heterogeneous responses of hours across households can account for the cyclicality of real wages in the data. I show that a monetary expansion increases average hourly earnings in the model because productive households increase hours worked more than low-skilled households. That is, earnings inequality increases in the wake of expansionary monetary policy shocks. Empirical evidence based on micro-level data supports this model's prediction (e.g., Dolado, Motyovszki and Pappa (2020)).[6] Related, another important finding is that household heterogeneity matters for the cyclicality of real wages conditional on a monetary policy shock: the representative-agent model fails to reconcile the mismatch, while the heterogeneous-agent counterpart can solve the puzzle. This finding is in line with Broer et al. (2019), who argue that a simple heterogeneous-agent model with price and wage rigidities is a more relevant benchmark setting for monetary policy analysis.

Related Literature

This paper is related to different strands of the literature that focus on the transmission mechanism of monetary policy in the presence of incomplete markets. Auclert (2017) argues that redistribution channels including earnings heterogeneity channels play an essential role in accounting for monetary policy effects on aggregate consumption. Kaplan, Moll and Violante (2018) consider a New Keynesian model with incomplete financial markets in which two types of assets are introduced with different degrees of liquidity and returns. Having a model's ability to reproduce empirically realistic wealth distributions across liquid and illiquid assets and a distribution of the marginal propensities to consume (MPC), they show that the indirect channels from general equilibrium ef-

[6] By using monthly data from the Current Population Survey (CPS), Dolado, Motyovszki and Pappa (2020) find that an expansionary monetary policy shock increases relative employment between high-skilled and less-skilled workers, and this result holds in various empirical specifications. I also replicate their results in Figure 4 in Section 3.3.

fects dominate the direct effects, which are mainly from intertemporal substitution effects. Werning (2015) also studies the monetary transmission mechanism in the presence of an incomplete market and finds that indirect channels offset direct effects. Gornemann, Kuester and Nakajima (2016) consider a heterogeneous-agent New Keynesian economy where households are different in their employment status and wealth. They show that contractionary monetary policy shocks increase inequality, and, importantly, a majority of households prefer substantial stabilization of unemployment even though this implies deviations from stable price dynamics. The main contribution in this paper relative to the previous studies in this literature is that i) the current study incorporates wage rigidities into an incomplete markets model in the context of New Keynesian economies to study the cyclicality of real wages conditional on monetary policy shocks, and ii) it also reconciles the puzzling mismatch between the data and the model's predictions.

This paper can complement the recent discussions on the importance of sticky wages in the monetary transmission channel. Perhaps most closely related to this paper is Broer et al. (2019), who study the interaction between inequality and monetary policy by using a tractable heterogeneous-agent version of the New Keynesian model. The key finding in Broer et al. (2019) is that whether nominal frictions arise from price or wage rigidity matters greatly for the transmission mechanism for monetary policy: wage rigidities are key to accounting for the plausible monetary transmission channel. Nekarda and Ramey (2020) empirically document that the price markup, which is based on the inverse of the labor share, increases in response to expansionary demand shocks, inconsistent with the standard sticky-price New Keynesian model's prediction. Nekarda and Ramey (2020) conclude that one possible way to solve the mismatch would be incorporating sticky wages as in the old Keynesian models. The main finding in this paper can back up this literature by showing that the true real wages are countercyclical conditional on a monetary policy, which implies that the monetary ministration mechanism in New Keynesian models with sticky wages is in accordance with the data.

Another important related work is Cantore, Ferroni and Leon-Ledesma (2020), who show robust cross-country evidence that the labor share is countercyclical and that real wages are procyclical conditional on a monetary policy shock. They argue that this empirical evidence cannot be consistent with any medium-scale New Keynesian models under consideration even if these models possibly break the close link between the labor share and the inverse markup. Accordingly, Cantore, Ferroni and Leon-Ledesma (2020) cast doubts on the standard monetary transmission mechanism in New Keynesian macroeconomic models where the labor share plays a key role. The current paper revisits this view. When taking household heterogeneity into account, the perceived mismatch can be addressed, and the disparity arises from an inconsistent measure for real wages between the data and the model. Moreover, in contrast to Cantore, Ferroni and Leon-Ledesma (2020)'s argument, the heterogeneous-agent New Keynesian model with sticky wages is able to replicate the joint behavior of real wages and the labor share if one uses the data-equivalent measure for real wages in the model.

The paper is organized as follows. Section 2 presents a quantitative New Keynesian model economy with heterogeneous households. Section 3 presents the primary results from the benchmark model, mainly focusing on the cyclical behavior of the labor share and real wages. Section 4 concludes the paper.

2 The Model

In this section, I present the economic environment of a quantitative New Keynesian model economy with heterogeneous households. The model economy builds on a heterogeneous-agent model with incomplete asset markets and nominal price and wage rigidities. The model economy has four building blocks: a continuum (measure one) of households with identical preferences but different productivity, firms, a central bank, and a government. In the model economy, labor productivity of individual households varies exogenously. However, households cannot fully insure against individual productivity shocks, which implies that asset markets are incomplete, as in Huggett (1993) and Aiyagari (1994). The market incompleteness, together with borrowing constraints, will generate substantial *ex-post* heterogeneity in households' wealth, income, and consumption, and it will also make households respond differently to aggregate risks. It is assumed that the government supplies public bonds to asset markets and collects taxes from households to finance interest payments for bonds. For the remaining part of the model, standard assumptions in the New Keynesian literature are employed: sticky nominal prices and wages, monopolistically competitive goods and labor markets, and a conventional Taylor rule.

2.1 Households

The model economy is populated by a continuum of infinitely lived households. Each household maximizes its expected lifetime utility by choosing consumption, c_t, and hours worked, h_t :

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \Xi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right) \right]$$

subject to

$$c_t + a_{t+1} = w_t z_t h_t + (1 + r_t) a_t - T_t + \xi_t, \tag{2}$$

and

$$a_{t+1} \geq \underline{a},$$

where $0 < \beta < 1$, $\Xi > 0$, and ν denote the time discount factor, a parameter for disutility from working, and labor supply elasticity, respectively. Each household is endowed with a unit of time in each period, allocated between hours worked and leisure. When a household supplies h_t units of labor, it earns $w_t z_t h_t$ as labor income, where w_t is the wage rate per effective unit of labor, and z_t denotes its labor productivity. Each household earns profit income, ξ_t, from firms and pays taxes, T_t, to the government. Labor productivity, z, is assumed to follow an AR(1) process in logs:

$$\ln z' = \rho_z \ln z + \varepsilon_z, \quad \varepsilon_z \sim N(0, {\sigma_z}^2).$$

Households can invest financial assets, a_t, on government bonds for a real rate of return, r_t. Following Huggett (1993) and Aiyagari (1994), the asset markets are incomplete: a_t is the only asset available to households to insure against idiosyncratic risks, z. A household faces a borrowing constraint that limits the fixed amount of debt: the assets holding, a_{t+1}, cannot go below $\underline{a}$ for all t.

It is assumed that nominal wages are rigid in the economy. To incorporate sticky wages, I follow Hagedorn et al. (2019) and assume that a union sells a different type of labor services, $z_t(k)h_t(k)$, provided by each household k, to a representative employment agency in a competitive market. The employment agency uses the differentiated labor services to produce aggregate effective labor input, N_t, according to the constant elasticity of substitution (CES) technology given by:

$$N_t = \left(\int_0^1 z_t(k) h_t(k)^{\frac{\epsilon_w - 1}{\epsilon_w}} dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{3}$$

where ϵ_w is the elasticity of substitution across labor services.

The employment agency maximizes profits given aggregate effective labor, N_t:

$$\max_{h_t(k)} \left\{ W_t N_t - \int_0^1 W_t(k) z_t(k) h_t(k) dk \right\}.$$

The profit maximization implies the labor demand curve for household k:

$$h_t(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\epsilon_w} N_t, \tag{4}$$

where W_t is the aggregate nominal wage, which can be defined as:

$$W_t = \left(\int_0^1 z_t(k) W_t(k)^{1-\epsilon_w} dk \right)^{\frac{1}{1-\epsilon_w}}. \tag{5}$$

Wage adjustment costs are introduced similar to a Rotemberg (1982)'s mechanism: the adjustment costs are given by a quadratic function of the change in nominal wages, governed by the parameter, $\theta_w \geq 0$, and are proportional to individual productivity, $z_t(j)$. The union sets the nominal wage W_t^* for an effective unit of labor such that $W_t(k) = W_t^*$ and $h_t(k) = h_t^*$ by solving the following problem:

$$\max_{W_t^*} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \frac{1}{1+r_s} \right) \left\{ \int_0^1 \left[\frac{W_t^*}{P_t} z_t(k) h_t^*(W_t^*, W_t, L_t) - \frac{g(h_t^*(W_t^*, W_t, N_t))}{C_t^{-\sigma}} \right] dk - \int_0^1 z_t(k) \frac{\theta_w}{2} \left(\frac{W_t^*}{W_{t-1}^*} - \overline{\Pi^w} \right)^2 N_t dk \right\}, \tag{6}$$

subject to

$$h_t(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\epsilon_w} N_t, \tag{7}$$

where $g(h) = \Xi \frac{h^{1+1/\nu}}{1+1/\nu}$, C_t is the aggregate consumption, and $\overline{\Pi^w}$ is the steady-state gross wage inflation. The first-order condition under $W_t^* = W_t$ and $h_t^* = H_t$ implies the standard wage Phillips curve:

$$w_t(\epsilon_w - 1) + \theta_w \left(\Pi_t^w - \overline{\Pi^w}\right)\Pi_t^w - \epsilon_w \Xi C_t^{-\sigma} H_t^{1/\nu} = \theta_w \mathbb{E}_t \left[\frac{1}{1+r_t}\left\{\Pi_t^w - \overline{\Pi^w}\right\}\Pi_{t+1}^w \frac{N_{t+1}}{N_t}\right], \tag{8}$$

where $w_t = \frac{W_t}{P_t}$, $\Pi_t^w = \frac{W_t}{W_{t-1}}$, and H_t is aggregate hours.[7] Following Eggertsson and Singh (2019) and Hagedorn et al. (2019), I assume that wage adjustment costs are perceived by the union, and are taken into account only in their maximization problem; however the wage adjustment costs do not involve any physical costs.[8]

The household's problem can be recursively written as follows. Define x and X as the vectors of individual and aggregate state variables, respectively: $x \equiv (a, z)$ and $X \equiv (\mu, \eta)$, where $\mu(x)$ is the type distribution of households, and η denotes monetary policy shocks.[9] The value function for an individual household, denoted by $V(x, X)$, is defined as:

$$V(x, X) = \max_{c, a', h}\left\{\frac{c^{1-\sigma}-1}{1-\sigma} - \Xi\frac{h^{1+1/\nu}}{1+1/\nu} + \beta\mathbb{E}\left[V(x', X')|z, \eta\right]\right\} \tag{9}$$

subject to

$$c + a' = wzh + (1+r)a - T + \xi,\ a' \geq \underline{a},$$

and

$$\mu' = \mathbb{T}(X),$$

where $\mathbb{T}$ denotes the law of motion for μ, time subindices are suppressed to simplify notation, and primes denote variables in the next period.

2.2 The Representative Final Goods-producing Firm

It is assumed that the representative finished goods-producing firm operates in a competitive sector. The final good firm uses $y_t(j)$ units of each intermediate good $j \in [0, 1]$ to produce a homogeneous output, Y_t, according to the CES technology given by:

[7] Equation 8 yields a bias since the aggregation theorem does not hold in the incomplete market economy. I correct this bias by using a time-invariant number obtained from the steady-state equilibrium. For a more detailed discussion, see Appendix.

[8] As discussed in Eggertsson and Singh (2019), Rotemberg (1982) preferably interprets the adjustment costs of prices not as actual menu costs but as "the negative effect on the reputation of firms."

[9] Denote $\mathcal{A}$ and $\mathcal{Z}$ for sets of all possible realizations of a and z, respectively. Then, the measure $\mu(a, z)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{Z}$.

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \tag{10}$$

where $\epsilon_p > 1$ is the elasticity of substitution for intermediate goods. The final good firm in this sector takes the final-goods price, P_t, as given and purchased at the nominal price $p_t(j)$ for each of its inputs, where $p_t(j)$ is the price of the jth intermediate input. The profit maximization problem of the representative finished goods-producing firm is given by:

$$\max_{y_t(j)} \left\{ P_t Y_t - \int_0^1 p_t(j) y_t(j) dj \right\}, \tag{11}$$

subject to Equation 10. The first-order condition for the final-goods firm's problem, together with the zero-profit condition, implies that the demand for intermediate good j is given as:

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\epsilon_p} Y_t \quad \text{where } P_t = \left(\int_0^1 p_t(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}. \tag{12}$$

2.3 Intermediate Goods-producing Firm

There is a continuum of monopolistically competitive firms indexed by $j \in [0, 1]$, each of which produces a different type of intermediate good $y_t(j)$. Intermediate goods-producing firms use $n_t(j)$ units of effective labor in order to produce $y_t(j)$ units of intermediate good j, following a decreasing returns to scale (DRS) production function:

$$y_t(j) = n_t(j)^{1-\alpha} - \Omega,$$

where $1-\alpha$ is the degree of decreasing returns to labor, and $\Omega \geq 0$ is the fixed cost of production.[10] It should be noted that fixed costs in production break the direct link between the labor share and marginal costs and help generate the countercyclical behavior of the labor share.[11] It is assumed

[10] The DRS technology breaks a tight link between the dynamics of wages and the labor share, but key findings in this paper do not depend on the DRS assumption and still hold with the CRS production function.

[11] Fixed costs are not the only way to produce the countercyclical labor share. Without the fixed cost, the CRS production function with wage rigidities would make the labor share countercyclical.

that nominal prices are rigid in the economy, and price adjustment is subject to a Rotemberg (1982)'s price setting mechanism: each intermediate goods firm j faces a quadratic cost of adjusting their price governed by the parameter, $\theta_p \geq 0$. An intermediate goods-producing firm j maximizes its expected discounted profit by choosing its price $p_t(j)$:

$$\max_{p_{t+s}(j)} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \left(\prod_{i=0}^{s} \frac{1}{1+r_{t+i}} \right) \left\{ \left(\frac{p_{t+s}(j)}{P_{t+s}} - mc_{t+s} \right) y_{t+s}(j) - \frac{\theta_p}{2} \left(\frac{p_{t+s}(j)}{p_{t+s-1}(j)} - \overline{\Pi^p} \right)^2 Y_{t+s} \right\} \right], \tag{13}$$

subject to

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\epsilon_p} Y_t, \tag{14}$$

where mc_{t+s} is the real marginal cost of a unit of intermediate good, and $\overline{\Pi^p}$ is the steady-state gross inflation. Equation 14 is the demand for intermediate good j, driven by the final good firm's optimization. In the symmetric equilibrium conditions (i.e., $p_t(j) = P_t$ and $y_t(j) = Y_t$),[12] the first-order condition associated with the optimal price implies:

$$\epsilon_p - 1 + \theta_p \left(\Pi_t^p - \overline{\Pi^p} \right) \Pi_t^p - \epsilon_p mc_t = \theta_p \mathbb{E}_t \left[\frac{1}{1+r_t} \left\{ \Pi_t^p - \overline{\Pi^p} \right\} \Pi_{t+1}^p \frac{Y_{t+1}}{Y_t} \right], \tag{15}$$

where $\Pi_t^p = \frac{P_t}{P_{t-1}}$. Similar to the wage adjustment process, I follow Hagedorn et al. (2019) and assume that price adjustment costs do not correspond to resource costs of price changes, but firms behave *as if* they were in the maximization problem. Accordingly, the aggregate dividend, D_t, paid by the intermediate goods-producing firms is defined as:

$$D_t = Y_t - w_t N_t. \tag{16}$$

[12] All intermediate goods-producing firms choose the same price and produce the same quantity since they face the identical profit maximization problem.

2.4 Government and Central Bank

The government plays three roles in this economy: they i) collect taxes from households, ii) supply public bonds, and iii) distribute profits from intermediate goods-producing firms to households. I follow McKay, Nakamura and Steinsson (2016) and assume that the government levies taxes according to household's labor productivity, z_t :

$$T(z_t) = \tau_t z_t, \tag{17}$$

where τ_t is a tax rate, which is proportional to z_t.[13] Since individual labor productivity follows an exogenous process, this assumption does not distort households' optimal choices. The government issues bonds with real face value B_t, and they change taxes to finance interest payments on this public debt. Finally, following McKay, Nakamura and Steinsson (2016), I assume that the government collects taxes to run a balanced budget keeping a constant level of public debt each period (i.e., $B_t = \overline{B}$):

$$r_t \overline{B} = \int T_t(z_t) d\mu_t, \tag{18}$$

The government is also in charge of distributing monopoly profits from intermediate goods-producing firms to households. As in Kaplan, Moll and Violante (2018), I assume that dividend distribution is proportional to both asset holdings and productivity:

$$\xi_t(a_t, z_t) = \left\{ \psi \Delta_t^a + (1 - \psi)\, \Delta_t^z \right\} D_t, \tag{19}$$

where ψ is the fraction of profits for asset holdings, $\Delta_t^a = \frac{a_t}{\int a_t d\mu_t}$, and $\Delta_t^z = \frac{z_t}{\int z_t d\mu_t}$. How profits are distributed across households is not important in this model economy since rigid adjustments of wages make movements in profits small, so they do not generate large distributional effects.

The gross nominal interest on risk-free bonds, R_t, is assumed to follow a conventional Taylor rule:

[13] While McKay, Nakamura and Steinsson (2016) employ a non-linear tax system (a positive tax rate only for the highest productivity), I use a linear tax system where τ is proportional to z.

$$\ln R_t = \ln \overline{R} + \phi_\pi \left(\ln \Pi_t^p - \ln \overline{\Pi^p}\right) + \eta_t, \tag{20}$$

where $\overline{R}$ is the deterministic steady-state value of the gross nominal interest rate, and $\phi_\pi > 1$ is a coefficient on the inflation gap. η_t is monetary policy shocks, which follow an AR(1) process:

$$\eta' = \rho_\eta \eta + \varepsilon_\eta, \quad \varepsilon_\eta \sim N(0, {\sigma_\eta}^2).$$

Finally, the relationship between the real interest rate, the nominal interest rate, and inflation satisfies the Fisher relation:

$$\mathbb{E}_t \left[\frac{1}{1+r_t} \frac{R_t}{\Pi_{t+1}}\right] = 1. \tag{21}$$

2.5 Definition of Equilibrium

A recursive competitive equilibrium is a value function $V(x, X)$, a transition operator $\mathbb{T}(X)$, a set of policy functions $\{c(x, X)$,$a'(x, X)$,$h(x, X), n_j(X), p_j(X), y_j(X), h_k(X), W_k(X)\}$, and a set of prices $\{w(X), r(X), R(X)$,$P(X), W(X)\}$ such that:

1. Individual households' optimization: given $w(X)$ and $r(X)$, optimal decision rules $c(x, X)$, $a'(x, X)$, and $h(x, X)$ solve the Bellman equation, $V(x, X)$.

2. Intermediate goods firms' optimization: given $w(X)$, $r(X)$, and $P(X)$, the associated optimal decision rules are $n_j(X)$ and $p_j(X)$.

3. Final good firm's optimization: given a set of prices $P(X)$ and $p_j(X)$, the associated optimal decision rules are $y_j(X)$ and $Y(X)$.

4. The union's optimization: given $w(X)$, $r(X)$, and $W(X)$, the associated optimal decision rule is $W_k(X)$.

5. The employment agency's optimization: given a set of prices $W(X)$ and $W_k(X)$, the associated optimal decision rules are $h_k(X)$ and $N(X)$.

6. The gross nominal interest rate, $R(X)$, satisfies the Taylor rule (Equation 20).

7. The Fisher relation holds: $\mathbb{E}\left[\frac{1}{1+r(X)}\frac{R(X)}{\Pi^p(X')}\right]=1$.

8. Balanced budget of the government: $r(X)\overline{B}=\int T(x,X)d\mu$.

9. For all Ω,

   - (Labor market) $N(X)=\int zh(x,X)d\mu=\int n_j(X)dj$
   - (Bond market) $\overline{B}=\int a'(x,X)d\mu$
   - (Goods market) $Y(X)=C(X)$ where $Y(X)=N(X)^{1-\alpha}-\Omega$ and $C(X)=\int c(x,X)d\mu$.

10. Consistency of individual and aggregate behaviors: for all $A^0\subset\mathcal{A}$ and $Z^0\subset\mathcal{Z}$,

$$\mu'(A^0,Z^0)=\int_{A^0,Z^0}\left\{\int_{\mathcal{A},\mathcal{Z}}\mathbf{1}_{a'=a'(\omega,\Omega)}d\Gamma_z(z'|z)d\mu\right\}da'dz',$$

where $\Gamma_z(z'|z)$ is a transition probability distribution function for z.

2.6 Calibration

I calibrate the model economy based on the values of structural parameters from literature. The parameter values used in the model economy are summarized in Table 1.

I use a value for the risk aversion parameter, σ, equal to one. The Frish elasticity of labor supply, γ, is set to one as well. Theses are standard values in the business cycle and New Keynesian literature. The disutility parameter of working, Ξ, is set so that the steady-state hours are 0.233.[14] The time discount factor, β, is chosen to obtain a steady state value of the real interest rate of 2 percent annualized, as in McKay, Nakamura and Steinsson (2016). Regarding individual labor productivity shocks, I follow Debortoli and Gali (2018) and set $\rho_z=0.978$ and $\sigma_z=0.193$. This set of parameters implies that individual wages display an autoregressive coefficient of 0.914 and an innovation standard deviation of 0.258 at an annual frequency, which are similar to the estimates of Floden and Linde (2001). The shock process is approximated with a 17 states Markov chain approximated using the Tauchen (1986) method. The borrowing limit, $\underline{a}$, is simply chosen to be zero following McKay, Nakamura and Steinsson (2016) and Hagedorn et al. (2019).

The degree of decreasing returns to labor is set to 0.7, implying that $\alpha=0.3$. The production fixed cost, Ω, is set for intermediate goods firms to have zero profit in the steady state. The elasticities of substitution across households and intermediate goods, ϵ_w and ϵ_p, are set equal to 10. The

[14]This value is from the product of average hours conditional on working (1/3) and the long-run employment rate (70 percent).

Table 1: Parameters of the Model Economy

Parameter	Value	Description	Source/Target Moments
		Households	
β	0.97216	Time discount factor	Real return to bond
σ	1.0	Risk-aversion	Standard
Ξ	15.39	Disutility parameter	See text
γ	1.0	Labor supply elasticity	Standard
ρ_z	0.978	Persistence of z shocks	Debortoli and Gali (2018)
σ_z	0.193	Standard deviation of z shocks	Debortoli and Gali (2018)
$\underline{a}$	0	Borrowing limit	McKay, Nakamura and Steinsson (2016)
ϵ_w	10	Elasticity of substitution	Standard
θ_w	99.7	Wage adjustment cost	See text
		Firms	
$1-\alpha$	0.7	Degree of decreasing return	Standard
Ω	0.198	Production fixed cost	Zero profit
ϵ_p	10	Elasticity of substitution	Standard
θ_p	45.4	Price adjustment cost	See text
		Government and Monetary Authority	
ϕ_π	1.5	Weight on inflation	Standard
$\overline{B}/\overline{Y}$	1.4	Public debt to annual GDP	McKay, Nakamura and Steinsson (2016)
ψ	0.3	Fraction of profits for asset	Kaplan, Moll and Violante (2018)
$\overline{\Pi^w}$	1.0	Steady state gross W inflation	Standard
$\overline{\Pi^p}$	1.0	Steady state gross P inflation	Standard
$\overline{R}$	1.005	Steady state gross real interest	McKay, Nakamura and Steinsson (2016)
ρ_η	0.7	Persistence of MP shocks	Standard
$100 \times \sigma_\eta$	0.25	Standard deviation of MP shocks	Standard

Rotemberg adjustment cost parameters for nominal wages and price, θ_w and θ_p, are chosen for the corresponding Calvo stickiness parameters to be 0.75 and 0.65, respectively.[15] These parameter values are consistent with the estimates of Smets and Wouters (2007).

Following McKay, Nakamura and Steinsson (2016), I set debt to annual GDP to 1.4, and the tax rate τ_t is chosen for the government to run a balanced budget every period. As in Kaplan, Moll and Violante (2018), I assume that the fraction of profits for asset holdings, ψ, is the same as α, i.e., $\psi = \alpha = 0.3$. As mentioned earlier, wage rigidity generates relatively small movements in profits, so how they are distributed across households does not significantly affect the main findings in this paper. The Taylor rule coefficient of inflation, ϕ_π, is chosen to be 1.5. The steady-state gross inflations for wages and price, $\overline{\Pi^w}$ and $\overline{\Pi^p}$, are assumed to be one. Regarding the monetary policy

[15] Given a Calvo parameter λ_i, the Rotemberg adjustment cost parameter, θ_i, can be computed such that: $\theta_i = \frac{\lambda_i(\epsilon_i - 1)}{(1-\lambda_i)(1-\beta\lambda_i)}$ where $i = \{w, p\}$.

shocks, I choose $\rho_\eta = 0.7$ and $\sigma_\eta = 0.0025$. Given the value of standard deviation, σ_η, the annualized size of a typical monetary policy shock is 100 basis points.

3 Results

3.1 Sticky Price vs. Sticky Wages

Is the monetary transmission mechanism in the sticky-price or sticky wages New Keynesian economy consistent with the data? Does the sticky wages model exhibit more plausible responses than the sticky-price counterpart? To give a concrete answer to these questions, I compare two model economies: the model with flexible wage and the model with sticky wages. In the model with flexible wages, it is assumed that price is rigid, but nominal wages are flexible. Accordingly, in the sticky-price model, I assume that $\theta_w = 0$, and real wages are determined by the equilibrium condition in the labor market.

I compare the dynamics of the two models—the model with flexible wages and the model with sticky wages—in Figure 1, which shows the impulse responses of aggregate variables to a 100-basis-point (annualized) expansionary monetary policy shock for 20 quarters. In an economy where price is sticky but wages are flexible, the expansionary monetary policy increases the demand for the labor input and real wages. Accordingly, households increase hours by 0.1 percent, and output or consumption rises by 0.2 percent. Finally, a 100-basis-point (annualized) monetary expansion increases annualized inflation by 0.88 percent point.

Importantly, in sticky-price New Keynesian models, the key mechanism of an unexpected monetary shock on real economic activity is through the procyclical labor share (or the countercyclical markups or profits). As is well-known, the labor share (or the price markup) plays a crucial role in the monetary transmission mechanism in standard New Keynesian economies. In this sense, monetary policy shocks in models should affect the cyclical behavior of the labor share in ways that are consistent with the data. As the dashed line in Figure 1 shows, when prices are sticky but wages are not, the labor share rises (or profits decrease) in response to expansionary monetary policy shocks. However, the procyclical labor share that New Keynesian models with price rigidities predict is inconsistent with empirical findings in the literature, such as Cantore, Ferroni and Leon-Ledesma (2020) and Nekarda and Ramey (2020) among others.[16] Cantore, Ferroni and Leon-Ledesma (2020)

[16] Christiano, Eichenbaum and Evans (2005) also provide empirical evidence that profits are procyclical conditional on monetary policy shocks.

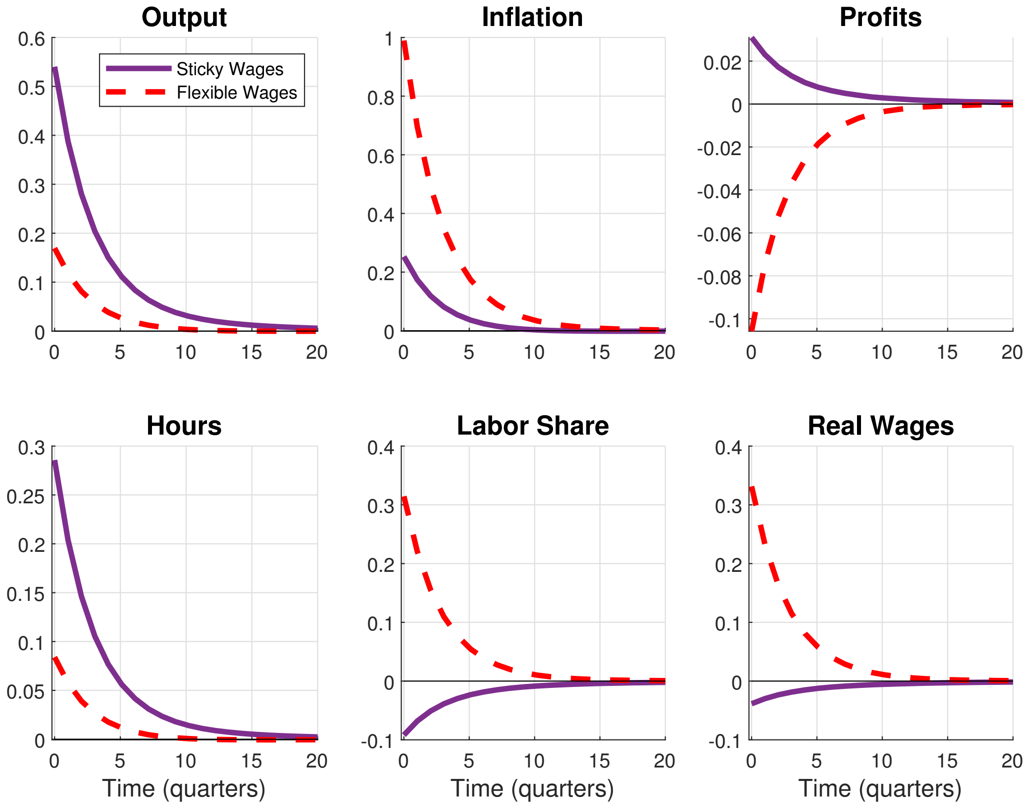

Figure 1: Impulse Responses of Aggregate Variables: Sticky Price vs. Sticky Wages
Note: Impulse response to a 100-basis-point (annualized) monetary policy shock. For output, consumption, hours, the labor share, and real wages, the y axis shows percent changes, while, for inflation and real rates , the y axis shows changes in annualized percentage points. The x-axis shows quarters after the shock.

empirically document that a contraction monetary policy tightening robustly induces an increase in the labor share in the U.S., the Euro area, the U.K., Australia, and Canada. Nekarda and Ramey (2020) also present evidence that the price markup, which is based on the inverse of the labor share, increases in response to expansionary monetary policy shocks. Thus, the monetary transmission mechanism in New Keynesian models with sticky-price only is not consistent with the data.

In response to the perceived mismatch between the data and predictions of the sticky-price New Keynesian model, Nekarda and Ramey (2020) suggest that one possible way to solve the mismatch would be incorporating sticky wages as in the old Keynesian models. Moreover, Broer et al. (2019) argue that wage rigidities are key to accounting for the plausible monetary transmission mechanism.[17] As shown in Figure 1 (solid lines), when wage rigidities are incorporated in the model economy, the transmission mechanism for monetary policy shocks is different from that in the model with flexible wages. Since the baseline calibration implies that prices are more flexible than wages,

[17]Using a medium-scale New Keynesian model, Christiano, Eichenbaum and Evans (2005) also show that sticky wages play an essential role in generating observed inertia in inflation and persistence in output in response to monetary policy shocks.

a monetary expansion decreases the real wage rate in the model economy with wage rigidities. The fall in real wages leads firms to increase their labor demand significantly,[18] which increases output (or consumption) relative to the model with a flexible wage setting. Sticky wages also make marginal costs faced by firms less affected by monetary policy given the production function, which strongly dampens the response of inflation relative to the economy with sticky price only.

The responses of aggregate variables in the economy with wage rigidities are in line with the data from a quantitative perspective: empirical evidence shows that i) monetary policy shocks have a more considerable impact on output in the calendar periods in which wages are relatively rigid (Olivei and Tenreyro, 2007, 2010; Bjorklund, Carlsson and Nordstrom Skans, 2019), and ii) inflation adjusts very little following a monetary policy shock (Christiano, Eichenbaum and Evans, 2005; Alpanda and Zubairy, 2017; Cantore, Ferroni and Leon-Ledesma, 2020). Notably, the monetary policy transmission mechanism in the economy with sticky wages is empirically realistic from a qualitative perspective: the sticky wages economy successfully generates the observed countercyclical labor share or procyclical profits. According to Figure 1, when wages are sticky, the labor share decreases while profits increase in response to a monetary policy easing. These results are in accordance with empirical findings in the aforementioned literature.

To summarize, the sticky-price New Keynesian economy with flexible wages generates counterfactual labor share dynamics in response to a monetary policy shock. In contrast, the model economy with sticky wages exhibits plausible responses of output, prices, the labor share, and profits to monetary policy shocks in quantitative and qualitative perspectives.

3.2 Are Real Wages Procyclical?

The main emphasis in this paper is on the cyclicality of real wages conditional on monetary policy shocks. In this subsection, I employ the quantitative model economy to revisit the cyclicality of real wages by comparing the dynamics of real wages (wage rate per effective unit of labor) and that of a data-consistent measure for real wages (average hourly earnings).

Since the key sticky-price transmission mechanism for monetary policy is at odds with the data, some researchers, such as Broer et al. (2019) and Nekarda and Ramey (2020), among others, argue that New Keynesian models may benefit from refocusing on wage rigidities rather than price rigidities. However, one of the critical limitations in the models with a standard sticky wage setting is that this class of models fails to jointly match the response of the labor share and real wages to mon-

[18] Note that the demand side solely determines aggregate effective labor in sticky wages models.

etary policy (Cantore, Ferroni and Leon-Ledesma, 2020). In the model economy with sticky wages, as the solid line in Figure 1 shows, real wages decrease in response to a monetary easing, which is not consistent with the data. Many papers in empirical literature provide robust evidence that the real wage rate is procyclical conditional on monetary policy shocks (Christiano, Eichenbaum and Evans, 2005; Coibion et al., 2017; Cantore, Ferroni and Leon-Ledesma, 2020).[19] Replicating the response of real wages is essential in studying the effects of monetary policy shocks since real wages are a crucial component of the labor share, through which a key monetary transmission channel in New Keynesian models works, as discussed above. In the model economy, the labor share, LS, can be defined as the ratio between real wages, w, and labor productivity, Y/N:

$$LS = \frac{wN}{Y} = \frac{w}{Y/N}, \tag{22}$$

where N denotes aggregate effective labor. Accordingly, real wages and productivity jointly determine the dynamics of the labor share. Importantly, it should be noted that w is defined as the wage rate per effective unit of labor in the model, as shown in the households' budget constraint (Equation 2). In practice, the model-consistent real wages may not be observable since effective labor is not an observable measure. In other words, in the data, one can observe total labor compensation, wN, but cannot identify w and N separately. Accordingly, the definition of real wages in the data cannot help being inconsistent with the model. Indeed, empirical literature uses an alternative definition of real wages. For example, Cantore, Ferroni and Leon-Ledesma (2020) construct the real wage measure for the U.S. economy as real Wages and Salaries from National Income and Product Accounts (NIPA) 1.12[20] divided by total hours worked in the economy from the Bureau of Labor Statistics (BLS). Recall that, in general, real wages in the data (denoted by ω) are defined as average hourly earnings—total labor compensation divided by total hours worked:

$$\omega = \frac{wN}{H}, \tag{23}$$

[19] In particular, Cantore, Ferroni and Leon-Ledesma (2020) provide richer cross-country empirical analysis on the effects of monetary policy on real wages and find that a monetary easing increases real wages during the Great Moderation period for all countries (the U.S., the Euro area, U.K., Australia, and Canada) under consideration. These facts are robust across time periods, shock identification methods, and information sets.

[20] Nominal wages and salaries are deflated by GDP Deflator to be transformed into real values.

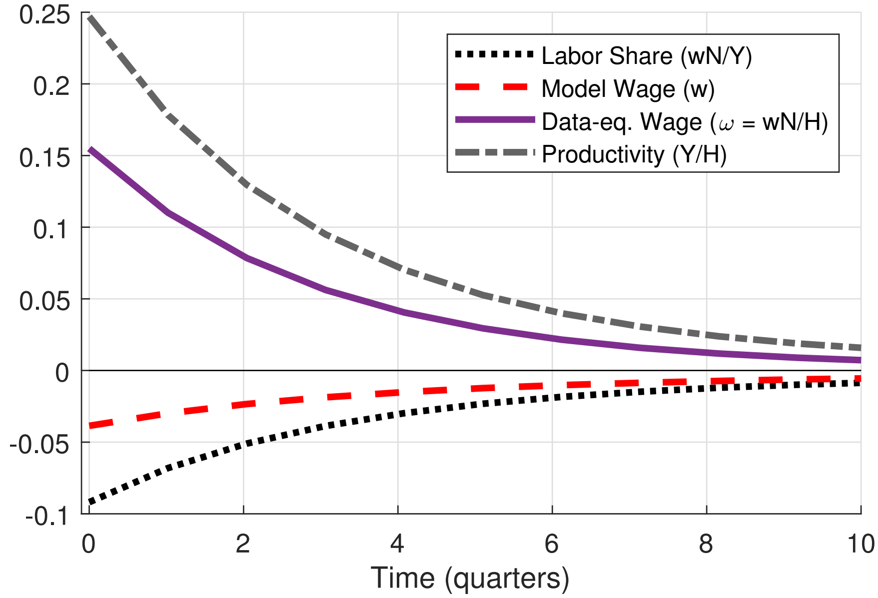

Figure 2: RESPONSE OF REAL WAGES: w VS. ω

Note: Impulse response to a 100-basis-point (annualized) monetary policy shock. The y axis shows percent changes while the x-axis shows quarters after the shock.

where H is aggregate hours in the economy. Based on Equation 23, data-equivalent real wages can be defined in the model economy as well. Of course, in the representative-agent model where there are infinitely many identical households, the dynamics of effective labor, N, will be the same as that of the aggregate hours H, so w has the same dynamics as ω all the time. In contrast, in heterogeneous-agent economies where households are different in their productivity, the dynamics of aggregate effective labor will be different from hours dynamics. Accordingly, the responses of w and ω may be very different *quantitatively* and/or *qualitatively*. The question is then, are data-consistent real wages also countercyclical conditional on monetary policy in the heterogeneous-agent New Keynesian model with sticky wages? This is an important question considering the previously-mentioned dependence of New Keynesian models on specific transmission channels of monetary policy shocks.

Figure 2 plots the responses of the two different real wages (w and ω) and other labor-related variables to a monetary policy shock in the model with benchmark calibration where both sticky price and wages are incorporated. An important finding is that in the benchmark model, even if the wage rate per effective unit of labor, w, falls following a monetary expansion, the data-equivalent measure for real wages, ω, indeed increases, which is consistent with the empirical findings in the literature discussed before. This finding implies that the true measure for real wages may be countercyclical conditional on monetary policy shocks, and data may predict the wrong direction of the real wage dynamics due to the measurement issue. Accordingly, the monetary transmission mechanism in New Keynesian models with wage rigidities is consistent with the data since true real wages are countercyclical conditional on a monetary policy shock.

This result contrasts with recent work by Cantore, Ferroni and Leon-Ledesma (2020). From rich empirical and theoretical analysis, Cantore, Ferroni and Leon-Ledesma (2020) conclude that medium-scale New Keynesian models commonly used for monetary policy study cannot generate empirically realistic responses of the labor share and real wages simultaneously. However, the above finding challenges this view: there is no puzzling mismatch between the data and the model if one uses the right measure, and the disparity between data and model is not from the model but the inconsistent definition of real wages in the data. Most importantly, the benchmark economy considered in this study is able to reproduce the joint behavior of real wages and the labor share if one uses average hourly earnings for real wages. Repeatedly, the data-equivalent measure for real wages increases, and the labor share decreases following a monetary expansion, which is in accordance with the data. In this sense, this finding can back up the recent discussions of the importance of wage rigidity in the transmission of monetary policy (Broer et al., 2019; Nekarda and Ramey, 2020): the predictions of New Keynesian economies with sticky wages are consistent with the data.

It is also important to note that household heterogeneity matters. As briefly discussed above, the representative-agent Keynesian counterpart predicts that both w and ω decrease following a monetary expansion. In this sense, the heterogeneous-agent New Keynesian model with wage rigidities can reconcile the puzzling mismatch between the data and predictions of the representative-agent model with sticky wages. This finding is in line with Broer et al. (2019). They suggest that a simple heterogeneous-agent model with sticky price and wages is a more relevant benchmark setting for monetary policy analysis since it provides better understanding of the micro-founded monetary transmission relative to a representative-agent counterpart.

Lastly, the model with benchmark calibration does an excellent job of accounting for the underlying behavior of the labor share dynamics. Empirical evidence provided by Christiano, Eichenbaum and Evans (2005) and Cantore, Ferroni and Leon-Ledesma (2020) shows that productivity should increase more than real wages to have the countercyclical labor share conditional on monetary policy shocks. It follows that the benchmark model can successfully reproduce the response of productivity, defined as output divided by total hours worked (i.e., Y/H), from the quantitative perspective. The labor share can be redefined as the ratio between $\omega\,(= wN/H)$ and productivity,

Y/H :

$$LS = \frac{wN/H}{Y/H} = \frac{\omega}{Y/H}. \tag{24}$$

According to Equation 24, Y/H should increase more than ω for the labor share to respond countercyclically to a monetary policy shock. Indeed, as shown in Figure 2, productivity increases by 0.25 percent, while ω increases by less (0.15 percent).

In summary, the heterogeneous-agent New Keynesian model with wage rigidities can jointly match the cyclical behavior of the labor share and data-consistent real wages conditional on monetary policy shocks. The perceived disparity between the data and theory arises due to an inconsistent measure for real wages in the data. In the model with benchmark calibration, the true real wage (wage rate per effective unit of labor) responds countercyclically to monetary expansion. In contrast, the data-consistent measure for real wages (average hourly earnings) increases. Therefore, I can conclude that true real wages may be countercyclical conditional on monetary policy shocks, which implies that the predictions of New Keynesian models with sticky wages are consistent with the data.

3.3 What Explains Procyclical Real Wages in Data?

At this point, the reader may wonder what explains procyclical real wages in the data. I answer this question by further decomposing the data-consistent measure for real wages, ω, in the model economy. According to Equation 23, given the negative response of w, effective labor per hours (N/H) should increase enough so that ω responds procyclically to a monetary policy shock. Indeed, it follows that N/H increases by around 0.2 percent (not shown in Figure 2), but ω increases by only 0.15 percent.

What does it mean? The positive response of effective labor per hours worked implies that skilled households increase hours of work more than unskilled households following expansionary monetary policy shocks. Figure 3 confirms this hypothesis: in the benchmark economy, relative hours between households in the highest productivity quintile (5th quintile) and the lowest productivity quintile (1st quintile) increase by 0.6 percent in the wake of a monetary expansion. Therefore, the heterogeneous responses of hours worked between high- and low-skilled households can account for the cyclicality of real wages in the data: an expansionary monetary shock increases average

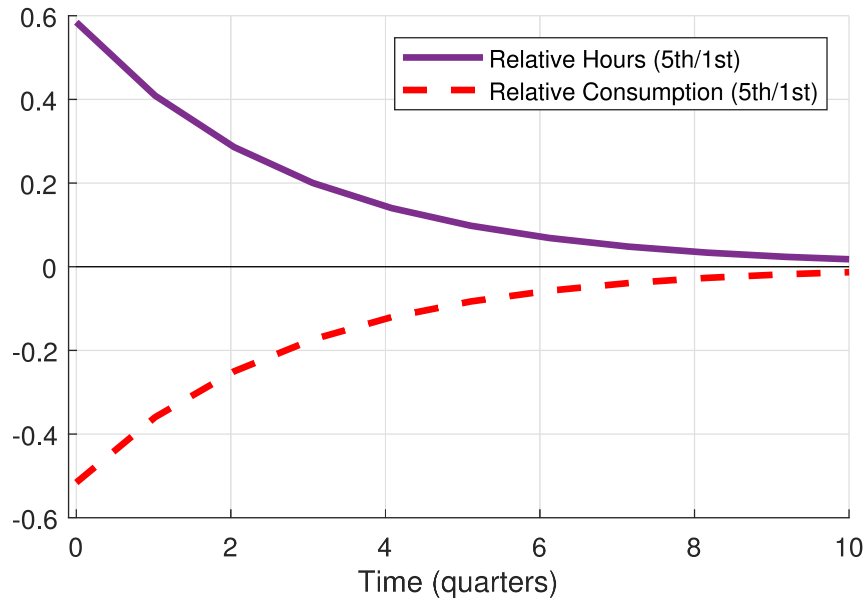

Figure 3: Response of Relative Hours and Consumption

Note: Impulse response to a 100-basis-point (annualized) monetary policy shock. The y axis shows percent changes while the x-axis shows quarters after the shock.

hourly earnings, ω, since productive households relatively work more, which leads to total labor compensation, wN, to increase more than total hours, H.

This model's prediction has empirical support from recent work by Dolado, Motyovszki and Pappa (2020). By using monthly data from the Current Population Survey (CPS), Dolado, Motyovszki and Pappa (2020) find that a monetary expansion increases relative employment between skilled and unskilled workers, and this result is robust to different empirical specifications. I replicate their results in Figure 4, which shows the responses of relative hours and employment between high and low-skilled workers. High-skilled and less-skilled workers are classified based on education status: whether they have experienced some college or not. Employment is the number of workers in each group while hours are defined as total hours worked for all persons in each skill category. Hence, the former can be interpreted as the extensive margin of labor supply, and the latter can be regarded as the total margin. Following Dolado, Motyovszki and Pappa (2020), I use the proxy VAR (Vector Autoregression) with monthly data covering the sample the period 1980:1 to 2007:12,[21] where measures developed by Romer and Romer (2004) are used as an external instrument for monetary policy shocks.[22] Consistent with findings in Dolado, Motyovszki and Pappa (2020), Figure 4 suggests that high-skilled workers increase total hours and employment more than

[21]As in Dolado, Motyovszki and Pappa (2020), I include five lags of each variable in the VAR system based on various information criteria.

[22]For this empirical analysis, as I follow closely Dolado, Motyovszki and Pappa (2020), I refer the readers to their paper for details on data and estimation specification.

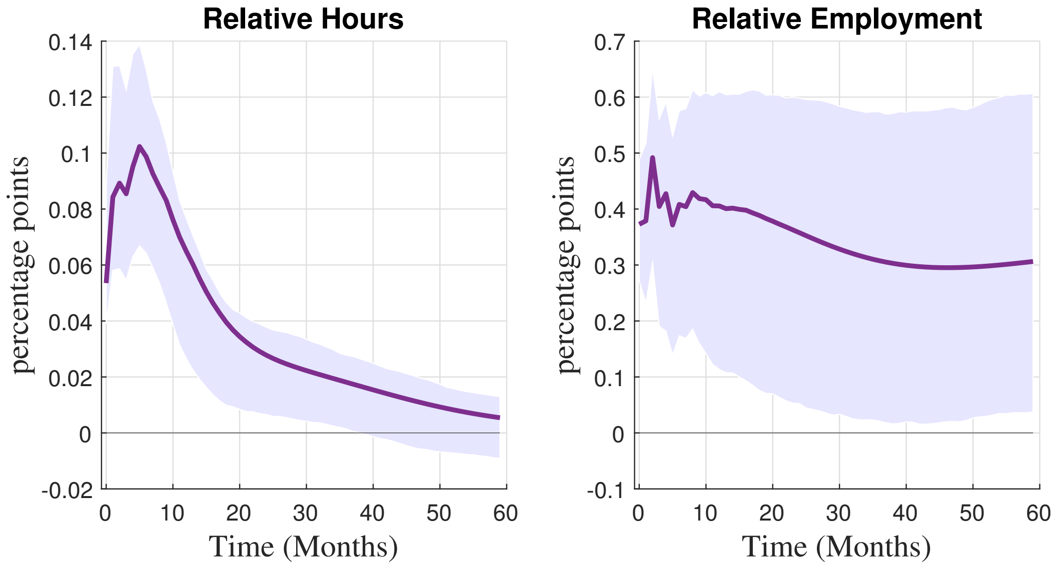

Figure 4: Responses of Relative Hours and Employment: Proxy VAR
Note: Impulse response to a 100-basis-point (annualized) monetary policy shock. The y axis shows percent point changes while the x-axis shows months after the shock. The shaded regions are the 68 percent confidence bands.

low-skilled workers in response to expansionary monetary policy shocks.[23] At the peaks, the relative hours worked increase by 0.1 percent point while the employment rate ratio increases by around 0.5 percentage points.[24] One may think that this empirical result is contrary to findings in Coibion et al. (2017).[25] However, Coibion et al. (2017) acknowledge that, even if countercyclical income and consumption inequality in response to monetary policy shocks appears to be robust, the response of earnings inequality is sensitive to the specific time period in the analysis. Indeed, they also report that earnings inequality is procyclical conditional on monetary policy shocks for some specifications.[26]

The question is then, why do productive households work more following a monetary expansion? This can be explained by the intratemporal optimality condition where households determine optimal hours and consumption. Since low-productivity households tend to a have higher marginal propensity to consume (MPC), they increase consumption more than households with higher productivity. Accordingly, as shown in Figure 3, the relative consumption between high-

[23] Intensive margin (relative hours per employed workers between high- and low-skilled workers) does not increase (not shown in the figure). Hence, the increase in relative total hours is driven by a significant rise in the extensive margin (relative employment).

[24] Following Dolado, Motyovszki and Pappa (2020), the two measures are not logged but times 100, so responses can be interpreted as percentage points.

[25] By using quarterly data from the Consumer Expenditure Survey (CEX), Coibion et al. (2017) study the effects of monetary policy shocks on various inequality (consumption, expenditure, income, and earnings).

[26] Cantore, Ferroni and Leon-Ledesma (2020) also discuss the possible effects of composition biases on real wages and provide evidence that wages of new hires increase more than aggregate wages in response to an expansionary monetary policy shock. They interpret this evidence as a result of entering low-wage workers during boom. However, according to Equation 23, their finding can be considered a result of an increase in hours or employment for high-skilled workers.

and low-productivity households decreases in response to expansionary monetary policy shocks.[27] Finally, given the same responses of real wages (w) and the labor markup across all households, smaller wealth effects from the little movement in consumption lead productive households to increase hours worked by more. Dolado, Motyovszki and Pappa (2020) also study this issue but provide a different story. Based on a New Keynesian model with capital-skill complementarity in production and asymmetric search and matching frictions, Dolado, Motyovszki and Pappa (2020) show that their model can predict an increase in labor income inequality following a monetary expansion. The increase in skilled employment results in more productive complementary capital, which leads to a further rise in investment demand and generates a multiplier effect.

To summarize, the data-consistent measure for real wages, ω, increases following a monetary expansion because productive households increase hours of work more than low-skilled households, and this model's prediction can be empirically supported.

3.4 Robustness

In this subsection, I conduct robustness analysis to see if the main findings are robust to different underlying primitives of the model economy: i) a different degree of wage rigidity (θ_w) and ii) a different parameter for profit distribution (ψ).

The upper panel of Figure 5 shows the responses of the labor share and the two wages (w and ω) with different degrees of wage rigidity. As discussed above, given a Calvo wage stickiness parameter λ_w, the Rotemberg adjustment cost parameter, θ_w, can be computed such that: $\theta_w = \frac{\lambda_w(\epsilon_w - 1)}{(1-\lambda_w)(1-\beta\lambda_w)}$. Recall that benchmark calibration is $\lambda_w = 0.75$ given that $\lambda_p = 0.65$. In this analysis, I include two additional cases: a degree of wage rigidity is smaller and larger than the benchmark case. Specifically, I consider two Calvo wage rigidity parameters: $\lambda_w = 0.65$, and $\lambda_w = 0.85$. According to the upper panel of Figure 5, as expected, the responses of the labor share and the wage per effective labor become larger as λ_w increases. Notably, the key findings in this paper are robust to different indexes of wage stickiness. Any values of λ_w show that the labor share and the true real wages decrease while the date-equivalent real wages increase following a monetary expansion.

The bottom panel of Figure 5 shows results with different parameters for profits distribution, ψ. As mentioned earlier, the main results do not depend on how profits are distributed across households since sticky wages produce a relatively small profit response (see Figure 1).

[27] The decrease in consumption inequality following a monetary expansion is well-supported by empirical work in Coibion et al. (2017).

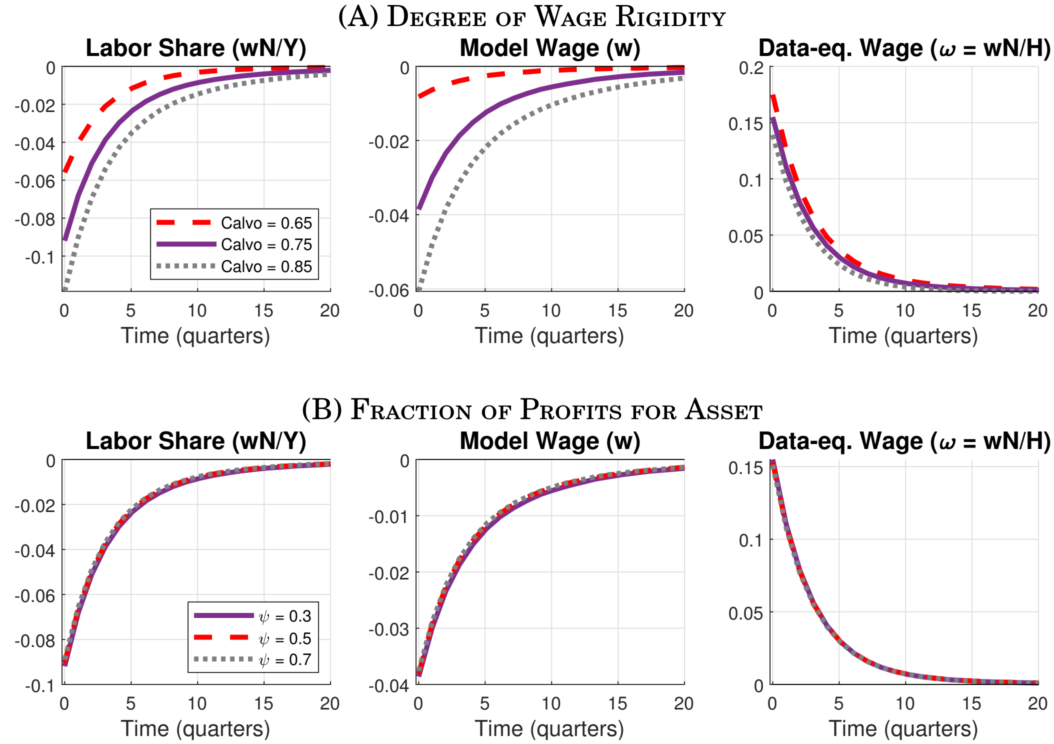

Figure 5: Robustness Analysis

Note: Impulse response to a 100-basis-point (annualized) monetary policy shock. The y axis shows percent changes while the x-axis shows months after the shock.

4 Conclusion

This paper revisits the conventional view on the cyclicality of real wages conditional on monetary policy shocks by developing a quantitative New Keynesian model economy with heterogeneous households. Existing papers in empirical literature provide robust evidence that real wages increase in response to expansionary monetary policy shocks (Christiano, Eichenbaum and Evans, 2005; Coibion et al., 2017; Cantore, Ferroni and Leon-Ledesma, 2020). In this sense, it is widely recognized that New Keynesian models with a standard sticky wage setting fail to jointly match the responses of the labor share and real wages to monetary policy shocks (Cantore, Ferroni and Leon-Ledesma, 2020). This paper challenges this view: there is no puzzling mismatch between the data and the model if the right measure for real wages is used. The disparity is not from the model but the inconsistent definition of real wages in the data.

From a quantitative analysis, I find that the heterogeneous-agent New Keynesian model with wage rigidities can jointly match the cyclical behavior of the labor share and data-consistent real wages conditional on monetary policy shocks. The perceived disparity between the data and theory arises due to an inconsistent measure for real wages in the data. In the model with benchmark calibration, the true real wage—wage rate per effective unit of labor—responds countercyclically to a monetary expansion while the data-consistent measure for real wages—total labor compensation divided by total hours worked—increases. Notably, the data-consistent real wages are procyclical conditional on a monetary policy shock since high-skilled households increase hours worked more than low-skilled households. This implies that earnings inequality increases following a monetary expansion. As a result, I argue that true real wages are countercyclical conditional on a monetary policy shock, which suggests that the predictions of New Keynesian models with sticky wages are consistent with the data.

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Appendix

A Computational Procedures

A.1 Steady-State Equilibrium

I summarize the computational algorithm used for the steady-state economy. In this step, I find the stationary measure, $\overline{\mu}$. The procedures are as follows.

Step 1. Have guesses for endogenous values such as β, Ξ, τ, and w.

Step 2. Construct grids for individual-state variables, such as asset holdings, a and logged individual labor productivity, $\hat{z} = \ln z$. N_a and N_z, denote the number of grids for a and z, respectively. I choose $N_a = 101$ and $N_z = 17$. The range of a is $[0, 40]$. More asset grid points are assigned on the lower range with a convex function. $\widehat{z}$ is equally spaced in the range of $[-3\sigma_{\hat{z}}, 3\sigma_{\hat{z}}]$, where $\sigma_{\hat{z}} = \sigma_z/\sqrt{1-\rho_z^2}$.

Step 3. Approximate the transition probability matrices for individual labor productivity, Γ_z, using Tauchen (1986).

Step 4. Solve the individual value functions at each grid point. In this step, I obtain the optimal decision rules for saving, $a'(a, z)$ and hours worked, $h(a, z)$, the value functions, $V(a, z)$. The detailed steps are as follows:

(a) Make an initial guess for the value function, $V_0(a, z)$ for every grid point.

(b) Solve the individual household's problem, and obtain $V_1(a, z)$:

$$V_1(a,z) = \max_{\{a',h\}} \left\{ \log\left(wzh + (1+r)a - T + \xi - a'\right) - \frac{\epsilon_w}{\epsilon_w - 1}\Xi\frac{h^{1+1/\nu}}{1+1/\nu} + \beta\sum_{z'=1}^{N_z}\Gamma_z(z'|z)V_0(a',z') \right\}$$

(c) If V_0 and V_1 are close enough for each grid point, go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (b).

Step 5. Obtain the time-invariant measure, $\overline{\mu}$, with finer grid points for a. Using cubic spline interpolation, compute the optimal decision rules for asset holdings with the new grid points. I compute $\overline{\mu}$ using the optimal decision rules with the finer grid points and transition probabilities for z, Γ_z.

Step 6. Compute aggregate variables using $\overline{\mu}$. If the aggregate values become sufficiently close to the targeted values, then the steady-state equilibrium of the economy is found. Otherwise, update the endogenous parameters, and go back to Step 4.

A.2 Equilibrium with Aggregate Fluctuations

I summarize the computational algorithm used for the model economy with monetary policy shocks. To solve the dynamic economy, the distribution across households, μ, which affects prices, should be tracked (Krusell and Smith, 1998). Since the first moment of assets is constant over time, I instead use the second moment of the distribution (variance) and the forecasting function to solve the economy with aggregate shocks.

Step 1. I construct grids for aggregate-state variables, such as money supply shocks and the variance of assets, and the individual-state variables such as the individual labor productivity and asset holdings. For the variance of an asset, Υ, and monetary policy shocks, η, I construct five grid points for both. For monetary policy shocks, I construct points in the range of $[-3\widetilde{\sigma}_\eta, 3\widetilde{\sigma}_\eta]$, where $\widetilde{\sigma}_\eta = \sigma_\eta / \sqrt{1-\rho_\eta^2}$. The grid points for Υ and η are equally spaced. The grids for individual-state variables are the same as those in the steady-state economy.

Step 3. I parameterize the forecasting functions for Υ', N, Π^w,Π^p, w, R, mc, and ς, where ς is the labor markup.

Step 4. Given the forecasting functions for Υ' r, w, and, ς, I solve the optimization problems for individual households. I obtain the policy functions for asset holdings, $a'(a, z, \Upsilon, \eta)$, and the hours decision rule, $h(a, z, \Upsilon, \eta)$.[28]

Step 5. I generate simulated data for 3,500 periods using the value functions obtained in Step 4. The details are as follows.

(a) I set the initial conditions for Υ, η, and $\mu(a, z)$.

(b) Given the forecasting functions, I compute the updated values for gross wage inflation Π^w, the interest rate, r, and labor markup, ς

(c) Update R, Π^p, mc, and w.

[28]As discussed earlier, the transition probabilities for z and η are approximated using Tauchen (1986).

(d) Given the forecasting functions, the evaluated value function obtained in Step 4, and the obtained new prices. I solve the optimization problems for individual households to get the policy functions for asset holdings, $a'(a, z)$, and the hours decision rule, $h(a, z)$.

(e) I compute aggregate variables using μ: $C = \int c(a, z)d\mu$, $N = \int zh(a, z)d\mu$, $\Upsilon' = \text{var}\{a'(a, z)\}$,[29] $H = \int h(a, z)d\mu$, and $Y = N^{1-\alpha} - \Omega$.

(f) Obtain the next period measure $\mu'(a, z)$ using $a'(a, z)$ and transition probabilities for z.

Step 6. I obtain the new coefficients for the forecasting functions by the OLS estimation using the simulated time series.[30] If the new coefficients are close enough to the previous ones, the simulation is done. Otherwise, I reset the coefficients, and go to Step 4.

B Bias Correction for Wage Phillips Curve

Equation 8 produces a bias since the aggregation theorem does not hold in an incomplete market economy, i.e., $w_t \neq \varsigma_t \Xi C_t^{-\sigma} H_t^{1/\upsilon}$, where ς_t is the labor markup. This is true even in the steady state. The steady-state real wages should be equal to the product of the labor wedge and the marginal rate of substitution between *individual* consumption, c, and hours, h:

$$w = \frac{\epsilon_w}{\epsilon_w - 1} \Xi c^{-\sigma} h^{1/\upsilon}. \tag{A.1}$$

If one plugs steady-state aggregate consumption and hours into Equation A.1, the equation has aggregation bias (i.e., $w \neq \frac{\epsilon_w}{\epsilon_w - 1} \Xi C^{-\sigma} H^{1/\upsilon}$). In the benchmark economy, the bias is 1.6 percent, so it is not negligible. Accordingly, to correct the New Keynesian Phillips Curve (Equation 8), I use time-invariant bias correction, obtained from the steady state condition. Bias correction, ζ, satisfies the following equation in the steady state:

$$w = \zeta \frac{\epsilon_w}{\epsilon_w - 1} \Xi C^{-\sigma} H^{1/\upsilon}. \tag{A.2}$$

C Cross-sectional Distributions

In this section, I discuss whether the model economy produces reasonable heterogeneity across households. Table A.1 compares the detailed information on income, earnings, and net asset hold-

[29] $\text{var}\{x\}$ denotes variance of a variable x.

[30] I drop the first 500 periods to eliminate the impact of the arbitrary choice of initial aggregate state variables.

Table A.1: Three Key Distributions

	Quintile					Gini
	1st	2nd	3rd	4th	5th	
U.S. Data						
Share of Income	2.18	6.63	11.80	19.47	59.91	0.57
Share of Earnings	-0.40	3.19	12.49	23.33	61.39	0.63
Share of Wealth	-0.39	1.74	5.72	13.43	79.49	0.78
Model Economy						
Share of Income	3.03	8.01	9.01	19.60	60.36	0.56
Share of Earnings	3.16	8.24	8.70	19.64	60.25	0.56
Share of Wealth	0.00	0.04	1.77	11.74	86.45	0.82

Note: Information for income and wealth in the data are from the Survey of Consumer Finances (SCF) 1992 in Diaz-Gimenez, Quadrini and Rios-Rull (1997).

ings in the model to the U.S data. The model economy reasonably replicates the income distribution of the U.S., making the income Gini index 0.55, which is comparable to the U.S. data (0.57). The earnings distribution in the model economy is less concentrated than the U.S. data: the model economy generates the earnings Gini index of about 0.56, while it is 0.63 in the U.S. data. Lastly, wealth inequality is well replicated by the model economy: the Gini index for wealth is 0.82 in the model, which is like what is observed in the U.S. data (0.78). Overall, the model economy successfully replicates empirically realistic heterogeneity across households.

D Distributional Implication

As in Broer et al. (2019), I examine distributional implications in model economies with different sources of nominal rigidities: sticky price and sticky wages. Distributional effects of monetary policy shocks are also an important issue in the literature (e.g., Gornemann, Kuester and Nakajima (2016) and Broer et al. (2019)). In principle, the key transmission channels in New Keynesian macroeconomic models operates through how wage income and firm's profits are distributed. Moreover, as discussed by Coibion et al. (2017) and Kaplan, Moll and Violante (2018), understanding of the transmission mechanism of monetary policy at the micro level is particularly important for the successful conduct of monetary policy. The model economy with benchmark calibration generates considerable heterogeneity across households,[31] so an unanticipated monetary policy may have siz-

[31] See Table A.1 in Appendix for the detailed information on distributions of income, earnings, and net asset holdings in the model and the U.S data.

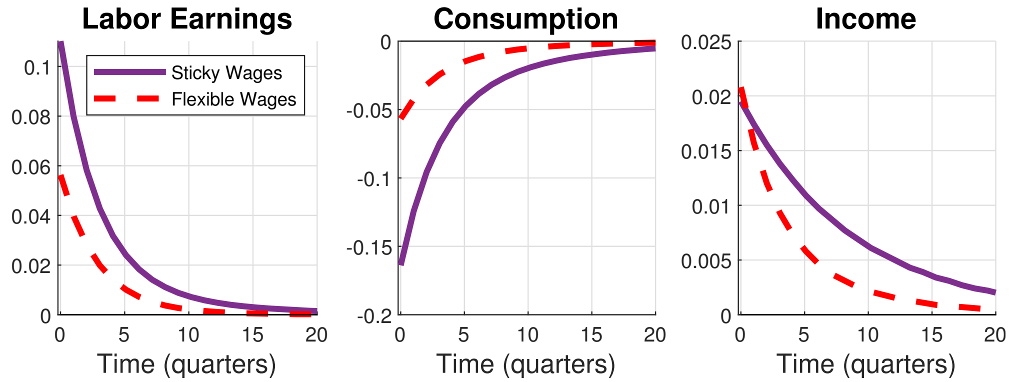

Figure A.1: Distributional Effects of Monetary Policy: Sticky Price vs. Sticky Wages
Note: Impulse response to a 100-basis-point (annualized) monetary policy shock. The y axis shows percent changes while the x-axis shows months after the shock.

able effects on distributions in the economy. I particularly discuss redistributive implications by asking if the distributional effects of monetary policy depends on the source of nominal frictions (price or wage rigidity). Similarly, Broer et al. (2019) also show that the aggregate and disaggregate effects of monetary policy shocks depend on the source of nominal frictions.

Figure A.1 depicts the effects of 100-basis-point (annualized) expansionary monetary policy shocks on Gini coefficients of labor earnings, consumption, and total income. As shown in Figure A.1, whether nominal frictions arise from price or wage rigidity matters for the effects of monetary policy shocks on inequality. In the model with sticky-price only, the distributional effect of monetary policy interventions is relatively small. The earnings Gini index increases by less than 0.06 percent while consumption inequality decreases by 0.05 percent. This is due to the substantial decrease in profits in the sticky-price model. The negative wealth effect leads to a small increase in consumption for poor households, making consumption inequality respond less. In turn, earnings inequality does not increase much, as the mechanism mentioned above from the intratemporal optimality condition. However, in the model with sticky wages, the distributional effect is larger due to the small movements in profits. In the sticky wages model, the earnings Gini coefficient increases by more than 0.1 percent following a monetary expansion, but the consumption Gini decreases by more than 0.15 percent.

There are a few things that should be discussed. First, one may think that the results here contrast with what Broer et al. (2019) find: they find that the distributional effect of monetary policy is large when only prices are sticky while it is smaller when rigid wages are added to the model. However, the definition of inequality is different between the two studies.[32] They focus on redis-

[32]Of course, the key assumptions are different across studies. To have analytical solutions, Broer et al. (2019) assume that households are of two types, workers and capitalists: only a small group of capitalists owns firms. However, in this

tributive effects between capitalists and workers rather than various dimensions of inequalities.[33] Second, unlike earnings and consumption inequalities, the source of nominal frictions does not matter for the effects on income inequality. As shown in Figure A.1, the on-impact responses of income Gini are similar in both models. Lastly, the responses of income inequality in both models are not consistent with the data from a qualitative perspective: empirical literature such as Coibion et al. (2017) and Furceri, Loungani and Zdzienicka (2018) suggests that a monetary expansion decreases income inequality. This is because the model economies here have only public bonds and do not include any physical capital. Hence, the definition of income in the model is different from that in the data.

paper, I assume that all households can work and own firms.

[33]Moreover, they do not explicitly compare the responses of inequalities between the model with different nominal rigidity sources. Instead, they conjecture the size of the distributional effect between the models based on dynamics of some aggregate variables. For example, they conclude that the model with price rigidity only may have a larger distributional effect since wage and profit incomes move in opposite directions.